# A METHOD OF OBTAINING STEADY STATE FLOWS OF GASEOUS MIXTURES WITH A GIVEN CONCENTRATION

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A method of obtaining a flow of a gaseous mixture with a given concentration through a supersonic nozzle is examined within the framework of nonviscous equilibrium flow; this method is based on measurements of pressures in a mixing chamber with separate outflow of the components.

In experiments with gases it is often necessary to create a flow of a gaseous mixture with a given concentration. If the discharges of gas are small or if their pressure is low, then the use of a previously prepared mixture does not eliminate the possible errors in the concentration. On sections of main gas pipelines, for which the Knudsen number Kn = $l/L$ has a magnitude of the order of 0.01 and more, it is possible for the separation of the components to be of such a nature that the mixture flows further, depleted of the heavy component, and the concentration of the original mixture varies with the passage of time.

The use of separate feed of the components to the mixing chamber required accurate measurements of the discharges of the individual gases.

For a case of outflow of gases from a certain mixing chamber through a Laval nozzle or a sonic nozzle in the case of supercritical pressure drops, the concentration of the gases can be accurately determined according to the magnitude of the pressure in the mixing chamber in the case of separate and combined flow of the gases, without having recourse to measurement of the discharge.

Since the analysis of this program is not known to us in the literature, a simple examination of it is given below with the following assumptions: a) the expansion of the gas is thermodynamically in equilibrium; b) the flow in the subcritical part of the nozzle is continuous; c) the influence of the viscosity can be neglected; d) the separation of the components in the nozzle before the critical cross section can also be neglected.

The mass discharge of any component in the case of its separate outflow

$$G_i = A_* p_{i0} \sqrt{\frac{\gamma_i m_i}{RT_0} \left(\frac{2}{\gamma_i + 1}\right)^{\frac{\gamma_i+1}{\gamma_i-1}}}. \tag{1}$$

Here $A_*$ is the area of the critical cross section of the nozzle; $p_0$ is the stagnation pressure; $T_0$ is the stagnation temperature.

The flow of the particles of the i-th component

$$N_i = A_* \frac{N_A}{\sqrt{m_i}} \cdot \frac{p_{i0}}{\sqrt{RT_0}} \sqrt{\gamma_i \left(\frac{2}{\gamma_i+1}\right)^{\frac{\gamma_i+1}{\gamma_i-1}}}. \tag{2}$$

Subsequently the parameters of the first and second components will be marked by the indices 1 and 2. The subscript c designates the parameter for the flow of the components in the mixture.

In the case of combined outflow of the gaseous mixture without separation, that is, with equality of the mean mass velocities of the component,

$$N_{ic} = n_{ic} u_c A. \tag{3}$$

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here n is the density of the particles; $u_c$ is the mean mass velocity of the mixture; A is the area of the flow cross section.

If the discharge of the components in the case of separate outflow is equal to the discharge in the case of combined outflow, then for a two-component mixture, since $N_1/N_2 = N_{1c}/N_{2c}$, it follows from (2) and (3) that

$$\frac{n_{1c}}{n_{2c}} = \frac{\sqrt{\gamma_1\left(\frac{2}{\gamma_1+1}\right)^{\frac{\gamma_1+1}{\gamma_1-1}}}}{\sqrt{\gamma_2\left(\frac{2}{\gamma_2+1}\right)^{\frac{\gamma_2+1}{\gamma_2-1}}}} \cdot \frac{p_{10}}{p_{20}} \sqrt{\frac{m_2}{m_1}}. \tag{4}$$

Since the molar concentration of the first component $f = (n_{1c}/n_{2c})/(1 + n_{1c}/n_{2c})$, then it is possible to write

$$f = \left[\sqrt{\frac{\gamma_1}{\gamma_2}\left[\left(\frac{2}{\gamma_1+1}\right)^{\frac{\gamma_1+1}{\gamma_1-1}} \Big/ \left(\frac{2}{\gamma_2+1}\right)^{\frac{\gamma_2+1}{\gamma_2-1}}\right]} \times \frac{p_{10}}{p_{20}}\sqrt{\frac{m_2}{m_1}}\right] \Bigg/ \left[1 + \sqrt{\frac{\gamma_1}{\gamma_2}\left[\left(\frac{2}{\gamma_1+1}\right)^{\frac{\gamma_1+1}{\gamma_1-1}} \Big/ \left(\frac{2}{\gamma_2+1}\right)^{\frac{\gamma_2+1}{\gamma_2-1}}\right]} \frac{p_{10}}{p_{20}}\sqrt{\frac{m_2}{m_1}}\right]. \tag{5}$$

The derived molecular mass of the gas mixture

$$m_c = m_1 f + m_2(1 - f). \tag{6}$$

For the case $\gamma_1 = \gamma_2 = \gamma_c$

$$\frac{n_{1c}}{n_{2c}} = \frac{p_{10}}{p_{20}}\sqrt{\frac{m_2}{m_1}}, \tag{7}$$

$$f = \frac{\frac{p_{10}}{p_{20}}\sqrt{\frac{m_2}{m_1}}}{1 + \frac{p_{10}}{p_{20}}\sqrt{\frac{m_2}{m_1}}}. \tag{8}$$

By using the relationships (4), (5) or, for gases of the same valency, (7) and (8), it is possible to select $p_{10}$ and $p_{20}$ which are necessary to obtain the desired concentration (for such a system of delivery of gases, in which the pressure in the mixing chamber will not influence the discharge of the components).

The magnitude of the pressure in the mixing chamber is of interest in the case of combined outflow of a binary mixture of gases from the nozzle; in separate outflow this has a pressure of $p_{10}$ and $p_{20}$ respectively in the mixing chamber.

By using the relationship (1) and expanding it to a mixture of gases from the condition $G_1 + G_2 = G_c$, we can write

$$\frac{p_{10}\sqrt{m_1}}{\sqrt{RT_0}}\sqrt{\gamma_1\left(\frac{2}{\gamma_1+1}\right)^{\frac{\gamma_1+1}{\gamma_1-1}}} + \frac{p_{20}\sqrt{m_2}}{\sqrt{RT_0}}\sqrt{\gamma_2\left(\frac{2}{\gamma_2+1}\right)^{\frac{\gamma_2+1}{\gamma_2-1}}} = \frac{p_{c0}\sqrt{m_c}}{\sqrt{RT_0}}\sqrt{\gamma_c\left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{\gamma_c-1}}}. \tag{9}$$

Hence, designating

$$\sqrt{\gamma_i\left(\frac{2}{\gamma_i+1}\right)^{\frac{\gamma_i+1}{\gamma_i-1}}} = K_i, \tag{10}$$

we obtain

$$p_{c0} = \frac{p_{10}\sqrt{m_1}K_1 + p_{20}\sqrt{m_2}K_2}{\sqrt{m_c}K_c}. \tag{11}$$

From (5) and (6) it follows that

$$m_c = \frac{p_{10}K_1\sqrt{m_1} + p_{20}K_2\sqrt{m_2}}{p_{10}K_1\frac{1}{\sqrt{m_1}} + p_{20}K_2\frac{1}{\sqrt{m_2}}}. \tag{12}$$

Substituting this expression into (11), we obtain after simplification

$$P_{c0} = \sqrt{\left(p_{10}\frac{K_1}{K_c} + p_{20}\frac{K_2}{K_c}\right)^2 + p_{10}p_{20}\frac{K_1K_2}{K_c^2}\frac{(\sqrt{m_1} - \sqrt{m_2})^2}{\sqrt{m_1m_2}}}; \tag{13}$$

the value $K_i$ depends only slightly on the variation of the adiabatic index $\gamma$. In the case of variation of $\gamma$ from 1.1 to 1.5, K increases by 1.115 times. Consequently, the value $p_{c0}$ is determined in a decisive manner by the values $p_{10}$ and $p_{20}$ and by the ratio of the molecular masses.

Where $\gamma_1 = \gamma_2 = \gamma_c$

$$p_{c0} = \sqrt{(p_{10} + p_{20})^2 + p_{10}p_{20}\frac{(\sqrt{m_1} - \sqrt{m_2})^2}{\sqrt{m_1m_2}}}. \tag{14}$$

It is evident that always in the case of $m_1 \neq m_2$ $p_{c0} > p_{10} + p_{20}$. Hence this difference can be significant. For example, for a mixture of argon with helium in the case of $p_{10} = p_{20}$ $p_{c0} = 1.17\ (p_{10} + p_{20})$.

The method of adjusting the discharge of gases through the nozzle is reduced to the following. If the discharges of gases are assigned according to the pressures $p_{01}$ and $p_{02}$ for separate outflow, it is possible to determine f according to formula (5) and then the value $p_{c0}$ according to formula (13); this value must be obtained in the case of combined feed. Nonagreement of the calculated values $p_{c0}$ with the measured value will indicate the infringement of one of the assumptions adopted at the beginning of the examination.

We will note yet another special feature of a flow of a gas mixture, which is important for certain calibration work. We will compare the concentrations of the components in the case of separate and combined flow in a channel when the discharges of the separate components are maintained. A simple analysis shows that for any cross section the relationship of the concentration of gases in the case of separate outflow and in the mixture is connected by the ratio

$$\frac{n_1}{n_2} = \frac{n_{1c}}{n_{2c}}\sqrt{\frac{m_1}{m_2}}. \tag{15}$$

For gases of the same valency

$$\frac{n_{1c}}{n_1} = \sqrt{f + \frac{m_2}{m_1}(1 - f)} \tag{16}$$

and

$$\frac{n_{2c}}{n_2} = \sqrt{\frac{m_1}{m_2}f + (1 - f)}. \tag{17}$$

The results of the above analysis can be used for calculating the parameters of a gas in the case of preparation of a mixture with a given concentration which flows through a supersonic nozzle.

## NOTATION

G is the mass discharge;
A is the area of the cross section of the nozzle;
p is the pressure;
T is the temperature;
m is the molecular mass;
$\gamma$ is the ratio of specific heats;
R is the universal gas constant;
$N_A$ is the Avogadro number;
N is the discharge of particles;
n is the density of particles;
u is the mean-mass velocity;
f is the molar concentration;

Kn is the Knudsen number;
$l$ is the length of the free path;
L is the characteristic dimension.

Subscripts

i is the arbitrary component;
* is the critical section;
0 is the stagnation parameter;
c is the mixture parameter;
1 and 2 are the first and second components.